# EXERCISE - 1

EXPRESS THEM IN THE FORM A + BI

**01.** $\dfrac{2 + 5i}{i(3 - 2i)}$

**02.** $\dfrac{(3 - i)^2}{2 - i}$

**03.** $\dfrac{i(4 + 6i)}{1 + i}$

**04.** $\dfrac{i(4 + 3i)}{1 - i}$

**05.** $\dfrac{-i(9 + 6i)}{2 - i}$

**06.** $\dfrac{(1 + 2i)(2 - 3i)}{3 + 4i}$

**07.** $\dfrac{2 + i}{(3 - i)(1 + 2i)}$

**08.** $\left(\dfrac{1}{1 - 2i} + \dfrac{3}{1 + i}\right)\left(\dfrac{3 + 4i}{2 - 4i}\right)$

**09.** $\left(1 + \dfrac{2}{i}\right)\left(3 + \dfrac{4}{i}\right)(5 + i)^{-1}$

**10.** $\left(1 + \dfrac{2}{i}\right)\left(1 + \dfrac{3}{i}\right)(1 + 4i)^{-1}$

**11.** $\dfrac{2 + 3i}{2 - 3i} + \dfrac{2 - 3i}{2 + 3i}$

**12.** $\dfrac{3 + 2i}{2 - 5i} + \dfrac{3 - 2i}{2 + 5i}$

**13.** $\dfrac{5 + 7i}{4 + 3i} + \dfrac{5 - 7i}{4 - 3i}$

**14.** $\dfrac{\sqrt{5} + i\sqrt{3}}{\sqrt{5} - i\sqrt{3}} + \dfrac{\sqrt{5} - i\sqrt{3}}{\sqrt{5} + i\sqrt{3}}$

**15.** Show that : $\left(\dfrac{\sqrt{7} + i\sqrt{3}}{\sqrt{7} - i\sqrt{3}} + \dfrac{\sqrt{7} - i\sqrt{3}}{\sqrt{7} + i\sqrt{3}}\right)$ is real

**16.** Show that :$z = \dfrac{5}{(1 - i)(2 - i)(3 - i)}$ is imaginary

**17.** $\left(\dfrac{1 + i}{\sqrt{2}}\right)^8 + \left(\dfrac{1 - i}{\sqrt{2}}\right)^8$

**18.** $\left(\dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}\right)^{10} + \left(\dfrac{1}{\sqrt{2}} - \dfrac{i}{\sqrt{2}}\right)^{10}$

**19.** if $a = \dfrac{-1 + \sqrt{3}i}{2}$ and $b = \dfrac{-1 - \sqrt{3}i}{2}$ Show that $a^2 = b$ and $b^2 = a$

**20.** $\left(3 + \dfrac{2}{i}\right)(i^6 - i^7)(1 + i^{11})$

**21.** $\dfrac{3i^5 + 2i^7 + i^9}{i^6 + 2i^8 + 3i^{18}}$

**22.** $\dfrac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2}$

# EXERCISE - 2

FIND THE VALUE OF

**01.** $x^3 + 2x^2 - 3x + 21$ ; $x = 1 + 2i$ **ans : 1**

**02.** $x^3 - x^2 + x + 46$ ; $x = 2 + 3i$ **ans : 7**

**03.** $x^4 - 4x^3 + 4x^2 + 8x + 43$ ; $x = 3 + 2i$ **ans : 4**

**04.** $x^3 + x^2 - x + 22$ ; $x = \frac{5}{1 - 2i}$ **ans : 7**

**05.** $x^3 - 5x^2 + 4x + 8$ ; $x = \frac{10}{3 - i}$ **ans : –2**

**06.** $2x^3 - 11x^2 + 44x + 27$ ; $x = \frac{25}{3 - 4i}$ **ans : 2**

**07.** $x^3 - x^2 - 3x + 30$ ; $x = \frac{9}{2 + \sqrt{5}i}$ **ans : 3**

**08.** $2x^4 + x^3 - x^2 - 24x - 24$ ; $x = 2i - 1$ **ans : 2i**

**09.** $2x^4 + 5x^3 + 7x^2 - x - 41$ ; $x = -2 - \sqrt{3}i$ **ans : –76**

**10.** $x^4 + 9x^3 + 35x^2 - x + 164$ ; $x = -5 + 2\sqrt{-4}$ **ans : 0**

**11.** $x^4 - 5x^3 + 12x^2 + 13x + 18$ ; $x = \frac{-1 + i\sqrt{3}}{2}$ **ans : √3i**

**12.** $x^4 - 4x^3 + 6x^2 + x - 13$ ; $x = \frac{4}{1 + i\sqrt{3}}$ **ans : –5√3i**

# EXERCISE - 3

SOLVE

**01.** $(x + 2y) + (2x - 3y)i + 4i = 5$ **ans : x = 1 ; y = 2**

**02.** $x(1 + 3i) + y(2 - i) - 5 + i^3 = 0$ ; find $x + y$ **ans : x = 1 ; y = 2**

**03.** $a + 2b + 15\, i^6 b = 7a + i^3(b + 6)$ ; find $a - b$ **ans : a =13 ; b = –6**

**04.** $x + 2i + 15i^6 y = 7x + i^3(y + 4)$ ; find $x + y$ **ans : x =15 ; y = –6**

**05.** $(i^4 + 3i)a + (i - 1)b + 5i^3 = 0$ ; find a , b **ans : a = b = 5⁄4**

**06.** $2x + i^9 y(2 + i) = xi^7 + 10i^{16}$ . **ans : x = 4 ; y = –2**

**07.** $\dfrac{x - 1}{1 + i} + \dfrac{y - 1}{1 - i} = i$ **ans : x = 0 ; y = 2**

**08.** $\dfrac{x}{1 + 2i} + \dfrac{y}{3 + 2i} = \dfrac{5 + 6i}{-1 + 8i}$ **ans : x = 1 ; y = 2**

**09.** $\dfrac{x + iy}{2 + 3i} + \dfrac{2 + i}{2 - 3i} = \dfrac{9(1 + i)}{13}$ **ans : x = 1 ; y = 2**

**10.** $\dfrac{x + iy}{4 - i} + \dfrac{2 + i}{4 + i} = \dfrac{26 - 15i}{17}$ **ans : x = 3 ; y = –5**

**11.** $\dfrac{xi^{16} - 3i^{27}}{2i^{40} + i^{37}y} = 1 + i^{99}$ ; find$(5x-7y)^2$ **ans : 0**

**12.** $\dfrac{x}{1 - 2i} + \dfrac{y}{1 + i} = \dfrac{3 + 2i}{-i}$ **ans : x = 5⁄3 ; y = –14⁄3**

**13.** $x+iy = (a+ib)^2$ ; show that : $x^2 + y^2 = (a^2 + b^2)^2$

**14.** $x + iy = \dfrac{a + ib}{a - ib}$ ; prove that : $x^2 + y^2 = 1$

**15.** $(x + iy)^3 = u + vi$ ; show that : $\dfrac{u}{x} + \dfrac{v}{y} = 4(x^2 - y^2)$

**16.** $(x + iy)^{1/3} = a + bi$ ; show that : $\dfrac{x}{a} + \dfrac{y}{b} = 4(a^2 - b^2)$

**17.** $\left(\dfrac{-1 + \sqrt{-3}}{2}\right)^3$ Prove it is a rational number

## EXERCISE - 4

FIND SQUARE ROOT OF

**01.** $-8 - 6i$ **ans : ± (1 – 3i)**

**02.** $1 + 4\sqrt{3}i$ **ans : ± (2 + √3i)**

| | | | | | |
|---|---|---|---|---|---|
| **03.** | $15 - 8i$ | **ans :** $\pm (4 - i)$ | **04.** | $3 - 4i$ | **ans :** $\pm (2 - i)$ |
| **05.** | $7 + 24i$ | **ans :** $\pm (4 + 3i)$ | **06.** | $6 + 8i$ | **ans :** $\pm \sqrt{2}(2 + i)$ |
| **07.** | $2(1 - \sqrt{3}i)$ | **ans :** $\pm (\sqrt{3} - i)$ | **08.** | $3 + 2\sqrt{10}i$ | **ans :** $\pm (\sqrt{5} + \sqrt{2}i)$ |
| **09.** | $2i$ | **ans :** $\pm (1 + i)$ | **10.** | $3i$ | **ans :** $\pm \sqrt{3/2}\,(1 + i)$ |

## EXERCISE - 5

EXPRESS THE FOLLOWING COMPLEX NUMBERS IN THE POLAR FORM

| | | |
|---|---|---|
| 01. | $1 + i$ | **ans :** $\sqrt{2}\left(\cos {}^{\pi}/_{4} + i \sin {}^{\pi}/_{4}\right)$ |
| 02. | $4 + 4\sqrt{3}\ I$ | **ans :** $8\left(\cos {}^{\pi}/_{3} + i \sin {}^{\pi}/_{3}\right)$ |
| 03. | $\frac{1 + \sqrt{3}\,i}{2}$ | **ans :** $1\left(\cos {}^{\pi}/_{3} + i \sin {}^{\pi}/_{3}\right)$ |
| 04. | $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ | **ans :** $1\left(\cos {}^{\pi}/_{4} + i \sin {}^{\pi}/_{4}\right)$ |
| 05. | $-1 + \sqrt{3}i$ | **ans :** $2\left(\cos {}^{2\pi}/_{3} + i \sin {}^{2\pi}/_{3}\right)$ |
| 06. | $\frac{1 + 2i}{1 - 3i}$ | **ans :** $\frac{1}{\sqrt{2}}\left(\cos {}^{3\pi}/_{4} + i \sin {}^{3\pi}/_{4}\right)$ |
| 07. | $\frac{1 + 7i}{(2 - i)^2}$ | **ans :** $\sqrt{2}\left(\cos {}^{3\pi}/_{4} + i \sin {}^{3\pi}/_{4}\right)$ |
| 08. | $\frac{1}{1 + i}$ | **ans :** $\frac{1}{\sqrt{2}}\left(\cos {}^{7\pi}/_{4} + i \sin {}^{7\pi}/_{4}\right)$ |
| 09. | $2i$ | **ans :** $2\left(\cos {}^{\pi}/_{2} + i \sin {}^{\pi}/_{2}\right)$ |
| 10. | $-3i$ | **ans :** $3\left(\cos {}^{3\pi}/_{2} + i \sin {}^{3\pi}/_{2}\right)$ |

## EXERCISE – 1

**01.** $\dfrac{2 + 5\,i}{i\,(3 - 2\,i)}$

**SOLUTION**

$$\frac{2 + 5i}{i\,(3 - 2i)}$$

$$= \frac{2 + 5i}{3i - 2i^2}$$

$$= \frac{2 + 5i}{3i + 2}$$

$$= \frac{2 + 5i}{3i + 2} \quad \frac{3i - 2}{3i - 2}$$

$$= \frac{(2 + 5i)(3i - 2)}{9i^2 - 4}$$

$$= \frac{6i - 4 + 15i^2 - 10i}{-9 - 4}$$

$$= \frac{6i - 4 - 15 - 10i}{-13}$$

$$= \frac{-19 - 4i}{-13}$$

$$= \frac{-19}{-13} + \frac{-4}{-13}\,i$$

$$= \frac{19}{13} + \frac{4\,i}{13}$$

**02.** $\dfrac{(3 - i)^2}{2 - i}$

**SOLUTION**

$$\frac{(3 - i)^2}{2 - i}$$

$$= \frac{9 - 6i + i^2}{2 - i}$$

$$= \frac{9 - 6i - 1}{2 - i}$$

$$= \frac{8 - 6i}{2 - i}$$

$$= \frac{8 - 6i}{2 - i} \quad \frac{2 + i}{2 + i}$$

$$= \frac{(8 - 6i)\,(2 + i)}{4 - i^2}$$

$$= \frac{16 + 8i - 12i - 6i^2}{4 + 1}$$

$$= \frac{16 + 8i - 12i + 6}{5}$$

$$= \frac{22 - 4i}{5}$$

$$= \frac{22}{5} + \frac{-\,4i}{5}$$

**03.** $\dfrac{i\,(4 + 6i)}{1 + i}$

**SOLUTION**

$$\frac{i\,(4 + 6i)}{1 + i}$$

$$= \frac{4i + 6i^2}{1 + i}$$

$$= \frac{4i - 6}{1 + i}$$

$$= \frac{4i - 6}{1 + i} \quad \frac{1 - i}{1 - i}$$

$$= \frac{(4i - 6)\,(1 - i)}{1 - i^2}$$

$$= \frac{4i - 4i^2 - 6 - 6i}{1 + 1}$$

$$= \frac{4i + 4 - 6 + 6i}{2}$$

$$= \frac{-2 + 10i}{2}$$

$$= -\,1 + 5i$$

**04.** $\dfrac{i\,(4+3i)}{1-i}$

**SOLUTION**

$$\frac{i\,(4+3i)}{1-i}$$

$$= \frac{4i+3i^2}{1-i}$$

$$= \frac{4i-3}{1-i}$$

$$= \frac{4i-3}{1-i}\quad\frac{1+i}{1+i}$$

$$= \frac{(4i-3)\,(1+i)}{1-i^2}$$

$$= \frac{4i+4i^2-3-3i}{1+1}$$

$$= \frac{4i-4-3-3i}{2}$$

$$= \frac{-7+1i}{2}$$

$$= \frac{-7}{2} + \frac{1}{2}\,i$$

**05.** $\dfrac{-\,i\,(9+6i)}{2-i}$

**SOLUTION**

$$\frac{-\,i\,(9+6i)}{2-i}$$

$$= \frac{-9i-6i^2}{2-i}$$

$$= \frac{-9i+6}{2-i}$$

$$= \frac{-9i+6}{2-i}\qquad\frac{2+i}{2+i}$$

$$= \frac{(-9i+6)\,(2+i)}{4-i^2}$$

$$= \frac{-18i-9i^2+12+6i}{4+1}$$

$$= \frac{-18i+9+12+6i}{2}$$

$$= \frac{21-12i}{5}$$

$$= \frac{21}{5} + \frac{-12}{5}\,i$$

**06.** $\dfrac{(1+2i)(2-3i)}{3+4i}$

**SOLUTION**

$$\frac{(1+2i)(2-3i)}{3+4i}$$

$$= \frac{2-3i+4i-6i^2}{3+4i}$$

$$= \frac{2-3i+4i+6}{3+4i}$$

$$= \frac{8+i}{3+4i}\quad\frac{3-4i}{3-4i}$$

$$= \frac{(8+i)(3-4i)}{9-16i^2}$$

$$= \frac{24-32i+3i-4i^2}{9+16}$$

$$= \frac{24-32i+3i+4}{25}$$

$$= \frac{28-29\,i}{25}$$

$$= \frac{28}{25} + \frac{-\,29}{25}\,i$$

**07.** $\dfrac{2+i}{(3-i)(1+2i)}$

**SOLUTION**

$$\frac{2+i}{(3-i)(1+2i)}$$

$$= \frac{2+i}{3+6i-i-2i^2}$$

$$= \frac{2+i}{3+6i-i+2}$$

$$= \frac{2+i}{5+5i}$$

$$= \frac{2+i}{5+5i} \quad \frac{5-5i}{5-5i}$$

$$= \frac{(2+i)(5-5i)}{25-25i^2}$$

$$= \frac{10-10i+5i-5i^2}{25+25}$$

$$= \frac{10-10i+5i+5}{50}$$

$$= \frac{15-5i}{50}$$

$$= \frac{15}{50} + \frac{-5i}{50}$$

$$= \frac{3}{10} + \frac{-1}{10} i$$

**08.** $\left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

**SOLUTION**

$$= \frac{1+i+3-6i}{(1-2i)(1+i)}\left(\frac{3+4i}{2-4i}\right)$$

$$= \frac{4-5i}{1+i-2i-2i^2}\left(\frac{3+4i}{2-4i}\right)$$

$$= \frac{4-5i}{1+i-2i+2}\left(\frac{3+4i}{2-4i}\right)$$

$$= \frac{4-5i}{3-i}\left(\frac{3+4i}{2-4i}\right)$$

$$= \frac{(4-5i)(3+4i)}{(3-i)\ (2-4i)}$$

$$= \frac{12+16i-15i-20i^2}{6-12i-2i+4i^2}$$

$$= \frac{12+16i-15i+20}{6-12i-2i-4}$$

$$= \frac{32+i}{2-14i}$$

$$= \frac{32+i}{2-14i} \quad \frac{2+14i}{2+14i}$$

$$= \frac{(32+i)\ (2+14i)}{(2-14i)\ (2-14i)}$$

$$= \frac{64+448i+2i+14i^2}{4-196i^2}$$

$$= \frac{64+448i+2i-14}{4+196}$$

$$= \frac{50+450i}{200}$$

$$= \frac{50}{200} + \frac{450}{200} i$$

$$= \frac{1}{4} + \frac{9}{4} i$$

**09.** $\left(1 + \frac{2}{i}\right)\left(3 + \frac{4}{i}\right)(5+i)^{-1}$

**SOLUTION**

$$= \frac{i+2}{i} \quad \frac{3i+4}{i} \quad \frac{1}{5+i}$$

$$= \frac{(i+2)(3\,i+4)}{i^2} \quad \frac{1}{5+i}$$

$$= \frac{3i^2+4i+6i+8}{i^2} \quad \frac{1}{5+i}$$

$$= \frac{-3+10i+8}{-1} \quad \frac{1}{5+i}$$

$$= \frac{5+10i}{-5-i}$$

$$= \frac{5+10i}{-5-i} \quad \frac{-5+i}{-5+i}$$

$$= \frac{(5 + 10i)\ (-5 + i)}{25 - i^2}$$

$$= \frac{-25 + 5i - 50i + 10i^2}{25 + 1}$$

$$= \frac{-25 + 5i - 50i - 10}{26}$$

$$= \frac{-35 - 45i}{26}$$

$$= \frac{-35}{26} + \frac{-45}{26}\, i$$

**10.** $\left(1 + \frac{2}{i}\right)\left(1 + \frac{3}{i}\right)(1 + 4i)^{-1}$

**SOLUTION**

$$= \frac{i + 2}{i}\ \frac{i + 3}{i}\ \frac{1}{1 + 4i}$$

$$= \frac{i^2 + 2i + 3i + 6}{i^2}\ \frac{1}{1 + 4i}$$

$$= \frac{-1 + 5i + 6}{-1}\ \frac{1}{1 + 4i}$$

$$= \frac{5 + 5i}{-1 - 4i}$$

$$= \frac{5 + 5i}{-1 - 4i}\ \frac{-1 + 4i}{-1 + 4i}$$

$$= \frac{-5 + 20i - 5i + 20i^2}{1 - 16i^2}$$

$$= \frac{-5 + 20i - 5i - 20}{1 + 16}$$

$$= \frac{-25 + 15i}{17}$$

$$= \frac{-25}{17} + \frac{15}{17}\, i$$

**11.** $\frac{2 + 3i}{2 - 3i} + \frac{2 - 3i}{2 + 3i}$

**SOLUTION**

$$= \frac{(2 + 3i)^2 + (2 - 3i)^2}{(2 + 3i)\ (2 - 3i)}$$

$$= \frac{(4 + \cancel{12i} + 9i^2) + (4 - \cancel{12i} + 9i^2)}{4 - 9i^2}$$

$$= \frac{4 - 9 + 4 - 9}{4 + 9}$$

$$= \frac{8 - 18}{13}$$

$$= \frac{-10}{13} + 0i$$

**12.** $\frac{3 + 2i}{2 - 5i} + \frac{3 - 2i}{2 + 5i}$

**SOLUTION**

$$= \frac{(3 + 2i)(2 + 5i) + (3 - 2i)(2 - 5i)}{(2 - 5i)\ (2 + 5i)}$$

$$= \frac{(6 + 15i + 4i + 10i^2) + (6 - 15i - 4i + 10i^2)}{4 - 25i^2}$$

$$= \frac{6 + \cancel{19i} - 10 + 6 - \cancel{19i} - 10}{4 + 25}$$

$$= \frac{12 - 20}{29}$$

$$= \frac{-8}{29} + 0i$$

**13.** $\frac{5 + 7i}{4 + 3i} + \frac{5 - 7i}{4 - 3i}$

**SOLUTION**

$$= \frac{(5 + 7i)(4 - 3i) + (5 - 7i)(4 + 3i)}{(4 + 3i)\ (4 - 3i)}$$

$$= \frac{(20 - 15i + 28i - 21i^2) + (20 + 15i - 28i - 21i^2)}{16 - 9i^2}$$

$$= \frac{20 + \cancel{13i} + 21 + 20 - \cancel{13i} - 21}{16 + 9}$$

$$= \frac{41 + 41}{25}$$

$$= \frac{82}{25} + 0i$$

**14.** $\dfrac{\sqrt{5} + i\sqrt{3}}{\sqrt{5} - i\sqrt{3}} + \dfrac{\sqrt{5} - i\sqrt{3}}{\sqrt{5} + i\sqrt{3}}$

**SOLUTION**

$$= \frac{\sqrt{5} + \sqrt{3}i}{\sqrt{5} - \sqrt{3}i} + \frac{\sqrt{5} - \sqrt{3}i}{\sqrt{5} + \sqrt{3}i}$$

$$= \frac{(\sqrt{5} + \sqrt{3}i)^2 + (\sqrt{5} - \sqrt{3}i)^2}{(\sqrt{5} - \sqrt{3}i)(\sqrt{5} + \sqrt{3}i)}$$

$$= \frac{(5 + 2\sqrt{15}i + 3i^2) + (5 - 2\sqrt{15}i + 3i^2)}{5 - 3i^2}$$

$$= \frac{5 + \cancel{2\sqrt{15}i} - 3 + 5 - \cancel{2\sqrt{15}i} - 3}{5 + 3}$$

$$= \frac{10 - 6}{8}$$

$$= \frac{4}{8}$$

$$= \frac{1}{2} + 0i$$

**15.** Show that :

$\dfrac{\sqrt{7} + i\sqrt{3}}{\sqrt{7} - i\sqrt{3}} + \dfrac{\sqrt{7} - i\sqrt{3}}{\sqrt{7} + i\sqrt{3}}$ is real

**SOLUTION**

$$= \frac{\sqrt{7} + \sqrt{3}i}{\sqrt{7} - \sqrt{3}i} + \frac{\sqrt{7} - \sqrt{3}i}{\sqrt{7} + \sqrt{3}i}$$

$$= \frac{(\sqrt{7} + \sqrt{3}i)^2 + (\sqrt{7} - \sqrt{3}i)^2}{(\sqrt{7} - \sqrt{3}i)(\sqrt{7} + \sqrt{3}i)}$$

$$= \frac{(7 + 2\sqrt{21}i + 3i^2) + (7 - 2\sqrt{21}i + 3i^2)}{7 - 3i^2}$$

$$= \frac{7 + \cancel{2\sqrt{21}i} - 3 + 7 - \cancel{2\sqrt{21}i} - 3}{7 + 3}$$

$$= \frac{14 - 6}{10}$$

$$= \frac{8}{10}$$

$$= \frac{4}{5} + 0i$$

**16.** Show that : $z = \dfrac{5}{(1 - i)(2 - i)(3 - i)}$ is imaginary

**SOLUTION**

$$\frac{5}{(1 - i)(2 - i)(3 - i)}$$

$$= \frac{5}{(1 - i)(6 - 2i - 3i + i^2)}$$

$$= \frac{5}{(1 - i)(6 - 5i - 1)}$$

$$= \frac{5}{(1 - i)(5 - 5i)}$$

$$= \frac{5}{(1 - i)5(1 - i)}$$

$$= \frac{1}{(1 - i)^2}$$

$$= \frac{1}{1 - 2i + i^2}$$

$$= \frac{1}{1 - 2i - 1}$$

$$= \frac{1}{-2i}\ \frac{i}{i}$$

$$= \frac{i}{-2i^2}$$

$$= \frac{i}{2}$$

$$= 0 + \frac{1}{2}i$$

**17.** $\left(\dfrac{1 + i}{\sqrt{2}}\right)^8 + \left(\dfrac{1 - i}{\sqrt{2}}\right)^8$

**SOLUTION**

$$= \left\{\left(\frac{1+i}{\sqrt{2}}\right)^2\right\}^4 + \left\{\left(\frac{1-i}{\sqrt{2}}\right)^2\right\}^4$$

$$= \left(\frac{1+2i+i^2}{2}\right)^4 + \left(\frac{1-2i+i^2}{2}\right)^4$$

$$= \left(\frac{1+2i-1}{2}\right)^4 + \left(\frac{1-2i-1}{2}\right)^4$$

$$= \left(\frac{2i}{2}\right)^4 + \left(\frac{-2i}{2}\right)^4$$

$$= (i)^4 + (-i)^4$$

$$= i^4 + i^4$$

$$= 2i^4$$

$$= 2(i^2)^2$$

$$= 2(-1)^2$$

$$= 2 + 0i$$

**18.** $\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{10} + \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{10}$

**SOLUTION**

$$= \left(\frac{1+i}{\sqrt{2}}\right)^{10} + \left(\frac{1-i}{\sqrt{2}}\right)^{10}$$

$$= \left\{\left(\frac{1+i}{\sqrt{2}}\right)^2\right\}^5 + \left\{\left(\frac{1-i}{\sqrt{2}}\right)^2\right\}^5$$

$$= \left(\frac{1+2i+i^2}{3}\right)^5 + \left(\frac{1-2i+i^2}{2}\right)^5$$

$$= \left(\frac{1+2i-1}{2}\right)^5 + \left(\frac{1-2i-1}{2}\right)^5$$

$$= \left(\frac{2i}{2}\right)^5 + \left(\frac{-2i}{2}\right)^5$$

$$= (i)^5 + (-i)^5$$

$$= i^5 - i^5$$

$$= 0$$

$$= 0 + 0i$$

**19.** if $a = \frac{-1+\sqrt{3}i}{2}$ and $b = \frac{-1-\sqrt{3}i}{2}$

Prove $a^2 = b$ and $b^2 = a$

**SOLUTION**

| $a^2$ | $b^2$ |
|---|---|
| $= \left(\frac{-1+\sqrt{3}i}{2}\right)^2$ | $= \left(\frac{-1-\sqrt{3}i}{2}\right)^2$ |
| $= \frac{1-2\sqrt{3}i+3i^2}{4}$ | $= \left(\frac{-(1+\sqrt{3}i)}{2}\right)^2$ |
| $= \frac{1-2\sqrt{3}i-3}{4}$ | $= \frac{1+2\sqrt{3}i+3i^2}{4}$ |
| $= \frac{-2-2\sqrt{3}i}{4}$ | $= \frac{1+2\sqrt{3}i-3}{4}$ |
| $= \frac{2(-1-\sqrt{3}i)}{4}$ | $= \frac{-2+2\sqrt{3}i}{4}$ |
| $= \frac{-1-\sqrt{3}i}{2}$ | $= \frac{2(-1+\sqrt{3}i)}{4}$ |
| $= b$ | $= \frac{-1+\sqrt{3}i}{2}$ |
| | $= a$ |

**20.** $\left(3 + \frac{2}{i}\right)(i^6 - i^7)(1 + i^{11})$

$i^6 = (i^2)^3 = (-1)^3 = -1$

$i^7 = i^6\, i = (i^2)^3 . i = (-1)^3 i = -i$

$i^{11} = i^{10}\, i = (i^2)^5 . i = (-1)^5 i = -i$

**BACK IN THE SUM**

$$= \left(3 + \frac{2}{i}\right)(-1 + i)(1 - i)$$

$= \dfrac{3i + 2}{i} \quad (-1 + i + i - i^2)$

$= \dfrac{3i + 2}{i} \quad (-1 + 2i + 1)$

$= \dfrac{3i + 2}{i} \quad 2i$

$= 2(2 + 3i)$

$= 4 + 6i$

**21.** $\dfrac{3i^5 + 2i^7 + i^9}{i^6 + 2i^8 + 3i^{18}}$

**SOLUTION**

$i^5 = i^4\, i = (i^2)^2 .i = (-1)^2 i = i$

$i^7 = i^6\, i = (i^2)^3 .i = (-1)^3 i = -i$

$i^9 = i^8\, i = (i^2)^4 .i = (-1)^4 i = i$

$i^6 = (i^2)^3 = (-1)^3 = -1$

$i^8 = (i^2)^4 = (-1)^4 = 1$

$i^{18} = (i^2)^9 = (-1)^9 = -1$

**BACK IN THE SUM**

$= \dfrac{3(i) + 2(-i) + i}{-1 + 2(1) + 3(-1)}$

$= \dfrac{3i - 2i + i}{-1 + 2 -3}$

$= \dfrac{2i}{-2}$

$= - i$

$= 0 + -1i$

**22.** $\dfrac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2}$

**SOLUTION**

$i^8 = (i^2)^4 = (-1)^4 = 1$

$i^{10} = (i^2)^5 = (-1)^5 = -1$

$i^9 = i^8\, i = (i^2)^4 .i = (-1)^4 i = i$

$i^{11} = i^{10}\, i = (i^2)^5 .i = (-1)^5 i = -i$

**BACK IN THE SUM**

$= \dfrac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2}$

$= \dfrac{4(1) - 3(i) + 3}{3(-i) - 4(-1) - 2}$

$= \dfrac{4 - 3i + 3}{-3i + 4 - 2}$

$= \dfrac{7 - 3i}{2 - 3i}$

$= \dfrac{7 - 3i}{2 - 3i} \quad \dfrac{2 + 3i}{2 + 3i}$

$= \dfrac{(7 - 3i)\ (2 + 3i)}{4 - 9i^2}$

$= \dfrac{14 + 21i - 6i - 9i^2}{4 + 9}$

$= \dfrac{14 + 15i + 9}{13}$

$= \dfrac{23 + 15i}{13}$

$= \dfrac{23}{13} + \dfrac{15\, i}{13}$

## EXERCISE – 2

**01.** $x^3 + 2x^2 - 3x + 21 \quad ; \quad x = 1 + 2i$

**SOLUTION**

$x = 1 + 2i$

$x - 1 = 2i$

**SQUARING**

$(x - 1)^2 = 4i^2$

$x^2 - 2x + 1 = -4$

$x^2 - 2x + 1 + 4 = 0$

$x^2 - 2x + 5 = 0$

$$\begin{array}{r|l} & \qquad x + 4 \\ \hline x^2 - 2x + 5 & x^3 + 2x^2 - 3x + 21 \\ & x^3 - 2x^2 + 5x \\ & \quad + \qquad - \\ \hline & \qquad 4x^2 - 8x + 21 \\ & \qquad 4x^2 - 8x + 20 \\ & \quad - \qquad + \qquad - \\ \hline & \qquad\qquad\qquad 1 \end{array}$$

**HENCE**

$x^3 + 2x^2 - 3x + 21$

$= (x^2 - 2x + 5)(x + 4) + 1$

$= 0(x + 4) + 1$

$= 0 + 1$

$= 1$

**02.** $x^3 - x^2 + x + 46 \quad ; \quad x = 2 + 3i$

**SOLUTION**

$x = 2 + 3i$

$x - 2 = 3i$

**SQUARING**

$(x - 2)^2 = 9i^2$

$x^2 - 4x + 4 = -9$

$x^2 - 4x + 4 + 9 = 0$

$x^2 + 4x + 13 = 0$

$$\begin{array}{r|l} & \qquad x + 3 \\ \hline x^2 + 4x + 13 & x^3 - x^2 + x + 46 \\ & x^3 + 4x^2 + 13x \\ & \quad + \qquad - \\ \hline & \qquad 3x^2 - 12x + 46 \\ & \qquad 3x^2 - 12x + 39 \\ & \quad - \qquad + \qquad - \\ \hline & \qquad\qquad\qquad 7 \end{array}$$

**HENCE**

$x^3 - x^2 + x + 46$

$= (x^2 + 4x + 13).(x + 3) + 7$

$= 0(x + 3) + 7$

$= 7$

**03.** $x^4 - 4x^3 + 4x^2 + 8x + 43 \quad ; \quad x = 3 + 2i$

**SOLUTION**

$x = 3 + 2i$

$x - 3 = 2i$

**SQUARING**

$(x - 3)^2 = 4i^2$

$x^2 - 6x + 9 = -4$

$x^2 - 6x + 9 + 4 = 0$

$x^2 - 6x + 13 = 0$

$$\begin{array}{r|l} & \qquad x^2 + 2x + 3 \\ \hline x^2 - 6x + 13 & x^4 - 4x^3 + 4x^2 + 8x + 43 \\ & x^4 - 6x^3 + 13x^2 \\ & \quad + \qquad - \\ \hline & \qquad 2x^3 - 9x^2 + 8x \\ & \qquad 2x^3 - 12x^2 + 26x \\ & \quad - \qquad + \qquad - \\ \hline & \qquad\qquad 3x^2 - 18x + 43 \\ & \qquad\qquad 3x^2 - 18x + 39 \\ & \qquad - \qquad + \qquad - \\ \hline & \qquad\qquad\qquad\qquad 4 \end{array}$$

**HENCE**

$x^4 - 4x^3 + 4x^2 + 8x + 43$

$= (x^2 - 6x + 13).(x^2 + 2x + 3) + 4$

$= 0(x^2 + 2x + 3) + 4$

$= 0 + 4$

$= 4$

**04.** $x^3 + x^2 - x + 22 \quad ; \quad x = \dfrac{5}{1 - 2i}$

**SOLUTION**

$$x = \frac{5}{1-2i}\,\frac{1+2i}{1+2i}$$

$$x = \frac{5(1+2i)}{1-4i^2}$$

$$x = \frac{5(1+2i)}{1+4}$$

$$x = \frac{5(1+2i)}{5}$$

$$x = 1 + 2i$$

$$x - 1 = 2i$$

$$(x-1)^2 = 4i^2$$

$$x^2 - 2x + 1 = -4$$

$$x^2 - 2x + 1 + 4 = 0$$

$$x^2 - 2x + 5 = 0$$

$$\begin{array}{r|llll}
 & x & + 3 & & \\
\hline
x^2 - 2x + 5 & x^3 & + x^2 & - x & + 22 \\
 & x^3 & - 2x^2 & + 5x & \\
 & + & & - & \\
\hline
 & & 3x^2 & - 6x & + 22 \\
 & & 3x^2 & - 6x & + 15 \\
 & & - & + & - \\
\hline
 & & & & 7
\end{array}$$

**HENCE**

$$x^3 + x^2 - x + 22$$

$$= (x^2 - 2x + 5).(x + 3) + 7$$

$$= 0(x + 3) + 7$$

$$= 7$$

**05.** $x^3 - 5x^2 + 4x + 8 \quad ; \quad x = \dfrac{10}{3 - i}$

**SOLUTION**

$$x = \frac{10}{3-i}\,\frac{3+i}{3+i}$$

$$x = \frac{10(3+i)}{9-i^2}$$

$$x = \frac{10(3+i)}{9+1}$$

$$x = \frac{10(3+i)}{10}$$

$$x = 3 + i$$

$$x - 3 = i$$

$$(x-3)^2 = i^2$$

$$x^2 - 6x + 9 = -1$$

$$x^2 - 6x + 9 + 1 = 0$$

$$x^2 - 6x + 10 = 0$$

$$\begin{array}{r|llll}
 & x & + 1 & & \\
\hline
x^2 - 6x + 10 & x^3 & - 5x^2 & + 4x & + 8 \\
 & x^3 & - 6x^2 & + 10x & \\
 & + & - & & \\
\hline
 & & x^2 & - 6x & + 8 \\
 & & x^2 & - 6x & + 10 \\
 & & - & + & - \\
\hline
 & & & & - 2
\end{array}$$

**HENCE**

$$x^3 - 5x^2 + 4x + 8$$

$$= (x^2 - 6x + 10).(x + 1) - 2$$

$$= 0(x + 1) - 2$$

$$= 0 - 2$$

$$= -2$$

**06.** $2x^3 - 11x^2 + 44x + 27 \quad ; x = \dfrac{25}{3 - 4i}$

**SOLUTION**

$$x = \frac{25}{3-4i}\,\frac{3+4i}{3+4i}$$

$$x = \frac{25(3+4i)}{9-16i^2}$$

$$x = \frac{25(3+4i)}{9+16}$$

$$x = \frac{25(3+4i)}{25}$$

$$x = 3 + 4i$$

$$x - 3 = 4i$$

$$(x-3)^2 = 16i^2$$

$$x^2 - 6x + 9 = -16$$

$$x^2 - 6x + 9 + 16 = 0$$

$$x^2 - 6x + 25 = 0$$

$$\begin{array}{r|llll}
 & 2x & + 1 & & \\
\hline
x^2 - 6x + 25 & 2x^3 & - 11x^2 & + 44x & + 27 \\
 & 2x^3 & - 12x^2 & + 50x & \\
 & + & & - & \\
\hline
 & & x^2 & - 6x & + 27 \\
 & & x^2 & - 6x & + 25 \\
 & & - & + & - \\
\hline
 & & & & 2
\end{array}$$

**HENCE**

$$2x^3 - 11x^2 + 44x + 27$$

$$= (x^2 - 6x + 25).(2x + 1) + 2$$

$$= 0(2x + 1) + 2$$

$$= 0 + 2$$

$$= 2$$

**07.** $x^3 - x^2 - 3x + 30 \quad ; \quad x = \dfrac{9}{2 + \sqrt{5}i}$

**SOLUTION**

$x = \dfrac{9}{2 + \sqrt{5}i}\,\dfrac{2 - \sqrt{5}i}{2 - \sqrt{5}i}$

$x = \dfrac{9(2 - \sqrt{5}i)}{4 - 5i^2}$

$x = \dfrac{25(2 - \sqrt{5}i)}{4 + 5}$

$x = \dfrac{9(2 - \sqrt{5}i)}{9}$

$x = 2 - \sqrt{5}i$

$x - 2 = -\sqrt{5}i$

$(x - 2)^2 = 5i^2$

$x^2 - 4x + 4 = -5$

$x^2 - 4x + 4 + 5 = 0$

$x^2 - 4x + 9 = 0$

$$\begin{array}{r|rrrr}
 & & x & +3 & \\ \hline
x^2 - 4x + 9 & x^3 & -x^2 & -3x & +30 \\
 & x^3 & -4x^2 & +9x & \\
 & & + & - & \\ \hline
 & & 3x^2 & -12x & +30 \\
 & & 3x^2 & -12x & +27 \\
 & & - & + & - \\ \hline
 & & & & 3
\end{array}$$

**HENCE**

$x^3 - x^2 - 3x + 30$

$= (x^2 - 4x + 9).(x + 3) + 3$

$= 0(x + 3) + 3$

$= 3$

**08.** $2x^4 + x^3 - x^2 - 24x - 24 \quad ; \quad x = 2i - 1$

**SOLUTION**

$x = 2i - 1$

$x + 1 = 2i$

**SQUARING**

$(x + 1)^2 = 4i^2$

$x^2 + 2x + 1 = -4$

$x^2 + 2x + 1 + 4 = 0$

$x^2 + 2x + 5 = 0$

$$\begin{array}{r|rrrrr}
 & & & 2x^2 & -3x & -5 \\ \hline
x^2 + 2x + 5 & 2x^4 & +x^3 & -x^2 & -24x & -24 \\
 & 2x^4 & +4x^3 & +10x^2 & & \\
 & - & & - & & \\ \hline
 & & -3x^3 & -11x^2 & -24x & \\
 & & -3x^3 & -6x^2 & -15x & \\
 & & + & + & + & \\ \hline
 & & & -5x^2 & -9x & -24 \\
 & & & -5x^2 & -10x & -25 \\
 & & & + & + & + \\ \hline
 & & & & x & +1
\end{array}$$

**HENCE**

$2x^4 + x^3 - x^2 - 24x - 24$

$= (x^2 + 2x + 5).(2x^2 - 3x - 5) + x + 1$

$= 0(2x^2 - 3x - 5) + x + 1$

$= x + 1$

$= 2i$

**09.** $2x^4 + 5x^3 + 7x^2 - x - 41 \quad ; x = -2 - \sqrt{3}i$

**SOLUTION**

$x = -2 - \sqrt{3}i$

$x + 2 = -\sqrt{3}i$

**SQUARING**

$(x + 2)^2 = 3i^2$

$x^2 + 4x + 4 = -3$

$x^2 + 4x + 7 = 0$

$$\begin{array}{r|rrrrr}
 & & & 2x^2 & -3x & +5 \\ \hline
x^2 + 4x + 7 & 2x^4 & +5x^3 & +7x^2 & -x & -41 \\
 & 2x^4 & +8x^3 & +14x^2 & & \\
 & - & & - & & \\ \hline
 & & -3x^3 & -7x^2 & -x & \\
 & & -3x^3 & -12x^2 & -21x & \\
 & & + & + & + & \\ \hline
 & & & +5x^2 & +20x & -41 \\
 & & & 5x^2 & +20x & +35 \\
 & & & - & - & - \\ \hline
 & & & & & -76
\end{array}$$

**HENCE**

$2x^4 + x^3 - x^2 - 24x - 24$

$= (x^2 + 4x + 7).(2x^2 - 3x + 5) - 76$

$= 0(2x^2 - 3x - 5) - 76$

$= -76$

**10.** $x^4 + 9x^3 + 35x^2 - x + 164 \quad ; x = -5 + 2\sqrt{-4}$

**SOLUTION**

$x = -5 + 2\sqrt{-4}$

$x + 5 = 2\sqrt{-4}$

**SQUARING**

$(x + 5)^2 = 4(-4)$

$x^2 + 10x + 25 = -16$

$x^2 + 10x + 41 = 0$

$$\begin{array}{r|l}
 & x^2 - x + 4 \\
\hline
x^2 + 10x + 41 & x^4 + 9x^3 + 35x^2 - x + 164 \\
 & x^4 + 10x^3 + 41x^2 \\
 & - \quad - \quad - \\
\hline
 & \quad - x^3 - 6x^2 - x \\
 & \quad - x^3 - 10x^2 - 41x \\
 & \quad + \quad + \quad + \\
\hline
 & \qquad 4x^2 + 40x + 164 \\
 & \qquad 4x^2 + 40x + 164 \\
 & \qquad + \quad + \quad + \\
\hline
 & \qquad\qquad 0
\end{array}$$

**HENCE**

$x^4 + 9x^3 + 35x^2 - x + 164$

$= (x^2 + 10x + 41).( x^2 - x + 4) + 0$

$= 0(x^2 - x + 4) + 0$

$= 0$

**11.** $x^4 - 5x^3 + 12x^2 + 13x + 18 \; ; \quad x = \frac{-1 + i\sqrt{3}}{2}$

**SOLUTION**

$x = \frac{-1 + i\sqrt{3}}{2}$

$2x = -1 + \sqrt{3}\, i$

$2x + 1 = \sqrt{3}i$

**SQUARING**

$(2x + 1)^2 = 3i^2$

$4x^2 + 4x + 1 = -3$

$4x^2 + 4x + 1 + 3 = 0$

$4x^2 + 4x + 4 = 0$

$x^2 + x + 1 = 0$

$$\begin{array}{r|l}
 & x^2 - 6x + 17 \\
\hline
x^2 + x + 1 & x^4 - 5x^3 + 12x^2 + 13x + 18 \\
 & x^4 + x^3 + x^2 \\
 & \quad - \quad - \\
\hline
 & \quad - 6x^3 + 11x^2 + 13x \\
 & \quad - 6x^3 - 6x^2 - 6x \\
 & \quad + \quad + \quad + \\
\hline
 & \qquad + 17x^2 + 19x + 18 \\
 & \qquad + 17x^2 + 17x + 17 \\
 & \qquad - \quad - \quad - \\
\hline
 & \qquad\qquad 2x + 1
\end{array}$$

**HENCE**

$x^4 - 5x^3 + 12x^2 + 13x + 18$

$= (x^2 + x + 1).( x^2 - 6x + 17) - 76$

$= 0(x^2 - 6x + 17) - 76$

$= 2x + 1$

$= \sqrt{3}i$

## EXERCISE – 3

**01.** $(x + 2y) + (2x - 3y)i + 4i = 5$

**SOLUTION**

$(x + 2y) + (2x - 3y)i = 5 - 4i$

**ON COMPARING**

2 x $\quad x + 2y = 5$

$2x - 3y = -4$

**SOLVING (1) & (2)**

$$\begin{array}{rcrcl} 2x & + & 4y & = & 10 \\ 2x & - & 3y & = & -4 \\ - & & + & & + \\ \hline & & 7y & = & 14 \\ & & y & = & 2 \end{array}$$

subs in (1)

$x + 2(2) = 5$

$x + 4 = 5$

$x = 5 - 4$

$x = 1$

**02.** $x(1 + 3i) + y(2 - i) - 5 + i^3 = 0$ ;find $x + y$

**SOLUTION**

$x(1 + 3i) + y(2 - i) - 5 + i^3 = 0$

$x + 3xi + 2y - yi - 5 - i = 0$

$(x + 2y - 5) + (3x - y - 1)i = 0 + 0i$

**ON COMPARING**

$x + 2y = 5$

2 x $\quad 3x - y = 1$

**SOLVING (1) & (2)**

$$\begin{array}{rcrcl} x & + & 2y & = & 5 \\ 6x & - & 2y & = & 2 \\ \hline 7x & & & = & 7 \\ & & x & = & 1 \end{array}$$

subs in (1)

$1 + 2y = 5$

$2y = 5 - 1$

$2y = 4$

$y = 2$

**03.** $a + 2b + 15\, i^6 b = 7a + i^3(b + 6)$ , $a - b$

**SOLUTION**

$i^6 = (i^2)^3 = (-1)^3 = -1$

$a + 2b + 15\,(-1)b = 7a + (-i)(b + 6)$

$a + 2b - 15b = 7a + -(b + 6)i$

$a - 13b + 0i = 7a + -(b + 6)i$

**ON COMPARING**

$$\begin{array}{l|l} a - 13b = 7a & -(b + 6) = 0 \\ -13b = 6a & -b - 6 = 0 \\ -13(-6) = 6a \longleftarrow & b = -6 \\ 13 = a & \end{array}$$

$\therefore\ a - b = 13 + 6 = 19$

**04.** $x + 2i + 15i^6 y = 7x + i^3(y + 4)$ ; find $x + y$

**SOLUTION**

$i^6 = (i^2)^3 = (-1)^3 = -1$

$x + 2i + 15\,(-1)y = 7x + (-i)(y + 4)$

$a + 2i - 15y = 7x + -(y + 4)i$

$x - 15y + 2i = 7x + -(y + 4)i$

**ON COMPARING**

$$\begin{array}{l|l} x - 15y = 7x & 2 = -(y + 4) \\ -15y = 6x & 2 = -y - 4 \\ -15(-6) = 6x \longleftarrow & 6 = -y \\ 15 = x & y = -6 \end{array}$$

$\therefore\ x + y = 15 - 6 = 9$

**05.** $(i^4 + 3i)a + (i - 1)b + 5i^3 = 0$

**SOLUTION**

$i^4 = (i^2)^2 = (-1)^2 = +1$

$(1 + 3i)a + (i - 1)b + 5(-i) = 0$

$a + 3ai + bi - b - 5i = 0$

$(a - b) + (3a + b - 5)i = 0 + 0i$

**ON COMPARING**

$$\begin{array}{rcll} a - b & = & 0 & \text{----} \quad (1) \\ 3a + b & = & 5 & \text{----} \quad (2) \\ \hline 4a & = & 5 & \\ a & = & \frac{5}{4} & \end{array}$$

subs in (1 )

$$b = \frac{5}{4}$$

**06.** $2x + i^9y(2 + i) = xi^7 + 10i^{16}$

**SOLUTION**

$i^7 = i^6\,i = (i^2)^3 .i = (-1)^3i = -i$

$i^9 = i^8\,i = (i^2)^4 .i = (-1)^4i = i$

$i^{16} = (i^2)^8 = (-1)^8 = +1$

**BACK IN THE SUM**

$2x + iy(2 + i) = x(-i) + 10(1)$

$2x + 2yi + yi^2 = -xi + 10$

$2x + 2yi - y = 10 - xi$

$2x - y + 2yi = 10 - xi$

**ON COMPARING**

$2x - y = 10$ ……… ( 1 )

$2y = -x$

2 x $x + 2y = 0$ ……… ( 2 )

**SOLVING (1) & (2)**

$$\begin{array}{rcl} 2x - y & = & 10 \\ 2x + 4y & = & 0 \\ - \quad - & & \\ \hline -5y & = & 10 \\ y & = & -2 \end{array}$$

subs in ( 2) $x + 2y = 0$

$x + 2(-2) = 0$

$x - 4 = 0$

$x = 4$

**07.** $\dfrac{x - 1}{1 + i} + \dfrac{y - 1}{1 - i} = i$

**SOLUTION**

$$\frac{(x - 1)\,(1 - i) + (y - 1)(1 + i)}{(1 + i)\,(1 - i)} = i$$

$$\frac{x - xi - 1 + i + y + yi - 1 - i}{1 - i^2} = i$$

$$\frac{x + y - 2 + (-x + y)i}{2} = i$$

**ON COMPARING**

$x + y - 2 + (-x + y)i = 0 + 2i$

$x + y - 2 = 0 \quad \therefore \quad x + y = 2$ …..(1)

$-x + y = 2 \quad \therefore \quad -x + y = 2$ ….. (2)

$$\begin{array}{rcl} 2y & = & 4 \\ y & = & 2 \ , \end{array}$$

subs in ( 1 ) $x = 0$

**08.** $\dfrac{x}{1 + 2i} + \dfrac{y}{3 + 2i} = \dfrac{5 + 6i}{-1 + 8i}$

**SOLUTION**

$$\frac{x(3 + 2i) + y(1 + 2i)}{(1 + 2i)\,(3 + 2i)} = \frac{5 + 6i}{-1 + 8i}$$

$$\frac{3x + 2xi + y + 2yi}{3 + 2i + 6i + 4i^2} = \frac{5 + 6i}{-1 + 8i}$$

$$\frac{3x + y + (2x + 2y)\,i}{3 + 2i + 6i - 4} = \frac{5 + 6i}{-1 + 8i}$$

$$\frac{3x + y + (2x + 2y)\,i}{-1 + 8i`} = \frac{5 + 6i}{-1 + 8i}$$

$$3x + y + (2x + 2y)\,i = 5 + 6i$$

$3x + y = 5$ ……… ( 1 )

$2x + 2y = 6$

$x + y = 3$ ……… ( 2 )

**SOLVING ( 1 ) & ( 2 )**

$$\begin{array}{r} 3x + y = 5 \\ x + y = 3 \\ \hline 2x = 2 \\ x = 1 \end{array}$$

subs in ( 2 ) $y = 2$

**09.** $$\frac{x + iy}{2 + 3i} + \frac{2 + i}{2 - 3i} = \frac{9(1 + i)}{13}$$

**SOLUTION**

$$\frac{(x + iy)(2 - 3i) + (2 + i)(2 + 3i)}{(2 + 3i)(2 - 3i)} = \frac{9 + 9i}{13}$$

$$\frac{2x - 3xi + 2yi - 3yi^2 + 4 + 6i + 2i + 3i^2}{4 - 9i^2} = \frac{9+9i}{13}$$

$$\frac{2x - 3xi + 2yi + 3y + 4 + 6i + 2i - 3}{4 + 9} = \frac{9 + 9i}{13}$$

$$\frac{2x - 3xi + 2yi + 3y + 1 + 8i}{13} = \frac{9 + 9i}{13}$$

$$(2x + 3y + 1) + (-3x + 2y + 8) = 9 + 9i$$

**ON COMPARING**

$2x + 3y + 1 = 9 \quad \therefore\ 2x + 3y = 8$ …… ( 1 )

$-3x + 2y + 8 = 9 \quad \therefore\ -3x + 2y = 1$ …… ( 2 )

$3 \times \quad 2x + 3y = 8 \qquad 6x + 9y = 24$

$2 \times \quad -3x + 2y = 1 \qquad -6x + 4y = 2$

$$\begin{array}{r} 13y = 26 \\ y = 2 \end{array}$$

subs in ( 1 )

$$\begin{array}{r} 2x + 6 = 8 \\ x = 1 \end{array}$$

**10.** $$\frac{x + iy}{4 - i} + \frac{2 + i}{4 + i} = \frac{26 - 15i}{17}$$

$$\frac{(x + iy)(4 + i) + (2 + i)(4 - i)}{(4 - i)(4 + i)} = \frac{26 - 15i}{17}$$

$$\frac{4x + xi + 4yi + yi^2 + 8 - 2i + 4i - i^2}{16 - i^2} = \frac{26-15i}{17}$$

$$\frac{4x + xi + 4yi - y + 8 - 2i + 4i + 1}{16 + 1} = \frac{26-15i}{17}$$

$$\frac{4x + xi + 4yi - y + 9 + 2i + 1}{17} = \frac{26-15i}{17}$$

$$(4x - y + 9) + (x + 4y + 2)i = 26 - 15i$$

**ON COMPARING**

$4x - y + 9 = 26 \quad \therefore\ 4x - y = 17$ ….(1)

$x + 4y + 2 = -15 \quad \therefore\ x + 4y = -17$ ….(2)

**SOLVING ( 1 ) & ( 2 )**

$$\begin{array}{r} 16x - 4y = 68 \\ x + 4y = -17 \\ \hline 17x = 51 \\ x = 3 \end{array}$$

subs in (1) $y = -5$

**11.** $$\frac{xi^{16} - 3i^{27}}{2i^{40} + i^{37}y} = 1 + i^{99}$$ ; find$(5x-7y)^2$

**SOLUTION**

$i^{16} = (i^2)^8 = (-1)^8 = 1$

$i^{40} = (i^2)^{20} = (-1)^{20} = 1$

$i^{27} = i^{26}\,i = (i^2)^{13}.i = (-1)^{13}i = -i$

$i^{37} = i^{36}\,i = (i^2)^{18}.i = (-1)^{18}i = i$

$i^{99} = i^{98}\,i = (i^2)^{49}.i = (-1)^{49}i = -i$

**BACK IN THE SUM**

$$\frac{x(1) - 3(-i)}{2(1) + iy} = 1 - i$$

$$\frac{x + 3i}{2 + yi} = 1 - i$$

$$x + 3i = (1 - i)(2 + yi)$$

$$x + 3i = 2 + yi - 2i - yi^2$$

$$x + 3i = 2 + yi - 2i + y$$

$$x + 3i = 2 + y + (y - 2)i$$

$x = 2 + y$ …….. ( 1 )

$3 = y - 2$ …….. ( 2 )

$\therefore\ y = 5$

subs in (1) $x = 7$

Hence $(5x-7y)^2$

$$= \left[5(7) - 7(5)\right]^2$$

$$= (35 - 35)^2$$

$$= 0$$

**12.** $$\frac{x}{1 - 2i} + \frac{y}{1 + i} = \frac{3 + 2i}{-i}$$

$$\frac{x(1 + i) + y(1 - 2i)}{(1 - 2i)(1 + i)} = \frac{3 + 2i}{-i}\ \frac{i}{i}$$

$$\frac{x + xi + y - 2yi}{1 + i - 2i - 2i^2} = \frac{3i + 2i^2}{-i^2}$$

$$\frac{x + y + (x - 2y)i}{1 + i - 2i + 2} = \frac{3i - 2}{1}$$

$$\frac{x + y + (x - 2y)i}{3 - i} = 3i - 2$$

$$x + y + (x - 2y)i = (3i - 2)(3 - i)$$

$$x + y + (x - 2y)i = 9i - 3i^2 - 6 + 2i$$

$$x + y + (x - 2y)i = 9i + 3 - 6 + 2i$$

$$x + y + (x - 2y)i = -3 + 11i$$

**ON COMPARING**

$$\begin{array}{rcr} x + y & = & -3 \\ x - 2y & = & 11 \\ - \quad + & & - \\ \hline 3y & = & -14 \end{array}$$

$$y = -\frac{14}{3}$$

subs in ( 1 )

$$x - \frac{14}{3} = -3$$

$$x = -3 + \frac{14}{3}$$

$$x = \frac{5}{3}$$

**13.** $x + iy = (a + ib)^2$

show that : $x^2 + y^2 = (a^2 + b^2)^2$

**SOLUTION**

$$x + yi = (a + ib)^2$$

$$x + yi = a^2 + 2abi + b^2i^2$$

$$x + yi = a^2 + 2abi - b^2$$

$$x + yi = a^2 - b^2 + 2abi$$

$x = a^2 - b^2$ & $y = 2ab$

**LHS**

$$= x^2 + y^2$$

$$= (a^2 - b^2)^2 + (2ab)^2$$

$$= a^4 - 2a^2b^2 + b^4 + 4a^2b^2$$

$$= a^4 + 2a^2b^2 + b^4$$

$$= (a^2 + b^2)^2$$

**= RHS**

**14.** $x + iy = \dfrac{a + ib}{a - ib}$ ;

prove that : $x^2 + y^2 = 1$

**SOLUTION**

$$x + iy = \frac{a + bi}{a - bi}\ \frac{a + bi}{a + bi}$$

$$x + yi = \frac{(a + bi)^2}{a^2 - b^2i^2}$$

$$x + yi = \frac{a^2 + 2abi + b^2i^2}{a^2 + b^2}$$

$$x + yi = \frac{a^2 + 2abi - b^2}{a^2 + b^2}$$

$$x + yi = \frac{a^2 - b^2}{a^2 + b^2} + \frac{2ab}{a^2 + b^2}\, i$$

**ON COMPARING**

$$x = \frac{a^2 - b^2}{a^2 + b^2} \quad \& \quad y = \frac{2ab}{a^2 + b^2}$$

**LHS**

$= x^2 + y^2$

$$= \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2 + \left(\frac{2ab}{a^2 + b^2}\right)^2$$

$$= \frac{(a^2 - b^2)^2}{(a^2 + b^2)^2} + \frac{4a^2b^2}{(a^2 + b^2)^2}$$

$$= \frac{a^4 - 2a^2b^2 + b^4 + 4a^2b^2}{(a^2 + b^2)^2}$$

$$= \frac{a^4 + 2a^2b^2 + b^4}{(a^2 + b^2)^2}$$

$$= \frac{(a^2 + b^2)^2}{(a^2 + b^2)^2}$$

$= 1$

**= RHS**

**15.** $(x + iy)^3 = u + vi$

show that : $\dfrac{u}{x} + \dfrac{v}{y} = 4(x^2 - y^2)$

**SOLUTION**

$$(x + iy)^3 = u + vi$$

$$x^3 + 3x^2yi + 3x(yi)^2 + (yi)^3 = u + vi$$

$$x^3 + 3x^2yi + 3xy^2i^2 + y^3i^3 = u + vi$$

$$x^3 + 3x^2yi + 3xy^2(-1) + y^3(-i) = u + vi$$

$$x^3 + 3x^2yi - 3xy^2 - y^3i = u + vi$$

$$(x^3 - 3xy^2) + (3x^2y - y^3)i = u + vi$$

**ON COMPARING**

$$u = x^3 - 3xy^2 \quad \& \quad v = 3x^2y - y^3$$

**LHS**

$$= \frac{u}{x} + \frac{v}{y}$$

$$= \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$$

$$= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$$

$= x^2 - 3y^3 + 3x^2 - y^2$

$= 4x^2 - 4y^2$

$= 4(x^2 - y^2)$

**= RHS**

**16.** $(x + iy)^{1/3} = a + bi$

show that : $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

**SOLUTION**

$(x + iy)^{1/3} = a + bi$

$x + yi = (a + bi)^3$

$x + yi = a^3 + 3a^2bi + 3a(bi)^2 + (bi)^3$

$x + yi = a^3 + 3a^2bi + 3ab^2i^2 + b^3i^3$

$x + yi = a^3 + 3a^2bi + 3ab^2(-1) + b^3(-i)$

$x + yi = a^3 + 3a^2bi - 3ab^2 - b^3i$

$x + yi = (a^3 - 3ab^2) + (3a^2b - b^3)i$

**ON COMPARING**

$x = a^3 - 3ab^2$ & $y = 3a^2b - b^3$

**LHS**

$= \frac{x}{a} + \frac{y}{b}$

$= \frac{a^3 - 3ab^2}{a} + \frac{3a^2b - b^3}{b}$

$= \frac{a(a^2 - 3b^2)}{a} + \frac{b(3a^2 - b^2)}{b}$

$= a^2 - 3b^3 + 3a^2 - b^2$

$= 4a^2 - 4b^2$

$= 4(a^2 - b^2)$

= **RHS**

**17.** $\left(\frac{-1+\sqrt{-3}}{2}\right)^3$ Prove it is a rational number

$\left(\frac{-1 + \sqrt{3}i}{2}\right)^3$

$= \frac{(-1)^3 + 3(-1)^2\sqrt{3}i + 3(-1)(\sqrt{3}i)^2 + (\sqrt{3}i)^3}{8}$

$= \frac{-1 + 3(1)\sqrt{3}i - 3.3i^2 + 3\sqrt{3}i^3}{8}$

$= \frac{-1 + 3\sqrt{3}i - 9(-1) + 3\sqrt{3}(-i)}{8}$

$= \frac{-1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i}{8}$

$= \frac{8}{8}$

= 1 HENCE RATIONAL

## EXERCISE – 4

**01.** $\sqrt{-8-6i}$

$\sqrt{-8-6i} = a + bi$

$-8 - 6i = (a + bi)^2$

$-8 - 6i = a^2 + 2abi + b^2i^2$

$-8 - 6i = a^2 + 2abi - b^2$

$-8 - 6i = a^2 - b^2 + 2abi$

**COMPARING**

$a^2 - b^2 = -8$ …….. (1)

$2ab = -6$

$ab = -3$

$b = \frac{-3}{a}$

subs in (1)

$a^2 - \left(\frac{-3}{a}\right)^2 = -8$

$a^2 - \frac{9}{a^2} = -8$

$\frac{a^4 - 9}{a^2} = -8$

$a^4 - 9 = -8a^2$

$a^4 + 8a^2 - 9 = 0$

$(a^2 + 9)(a^2 - 1) = 0$

$a^2 = 1$

$a = \pm 1$

**NOW**

$a = 1$

$b = \frac{-3}{a}$

$= \frac{-3}{1}$

$= -3$

**HENCE**

$\sqrt{-8-6i}$

$= 1 - 3i$

$a = -1$

$b = \frac{-3}{a}$

$= \frac{-3}{-1}$

$= 3$

**HENCE**

$\sqrt{-8-6i}$

$= -1 + 3i$

$\pm(1 - 3i)$

**02.** $\sqrt{-15-8i}$

$\sqrt{-15-8i} = a + bi$

$-15 - 8i = (a + bi)^2$

$-15 - 8i = a^2 + 2abi + b^2i^2$

$-15 - 8i = a^2 + 2abi - b^2$

$-15 - 8i = a^2 - b^2 + 2abi$

**COMPARING**

$a^2 - b^2 = -15$ …….. (1)

$2ab = -8$

$ab = -4$

$b = \frac{-4}{a}$

subs in (1)

$a^2 - \left(\frac{-4}{a}\right)^2 = -15$

$a^2 - \frac{16}{a^2} = -15$

$\frac{a^4 - 16}{a^2} = -15$

$a^4 - 16 = -15a^2$

$a^4 + 15a^2 - 16 = 0$

$(a^2 + 16)(a^2 - 1) = 0$

$a^2 = 1$

$a = \pm 1$

**NOW**

$a = 1$

$b = \frac{-4}{a}$

$= \frac{-4}{1}$

$= -4$

**HENCE**

$\sqrt{-15-8i}$

$= 1 - 4i$

$a = -1$

$b = \frac{-4}{a}$

$= \frac{-4}{-1}$

$= 4$

**HENCE**

$\sqrt{-15-8i}$

$= -1 + 4i$

$\pm(1 - 4i)$

**03.** $\sqrt{3 - 4i}$

$\sqrt{3 - 4i} = a + bi$

$3 - 4i = (a + bi)^2$

$3 - 4i = a^2 + 2abi + b^2i^2$

$3 - 4i = a^2 + 2abi - b^2$

$3 - 4i = a^2 - b^2 + 2abi$

**COMPARING**

$a^2 - b^2 = 3$ …….. (1)

$2ab = -4$

$ab = -2$

$b = \frac{-2}{a}$

subs in (1)

$a^2 - \left(\frac{-2}{a}\right)^2 = 3$

$a^2 - \frac{4}{a^2} = 3$

$\frac{a^4 - 4}{a^2} = 3$

$a^4 - 4 = 3a^2$

$a^4 - 3a^2 - 4 = 0$

$(a^2 - 4)(a^2 + 1) = 0$

$a^2 = 4$

$a = \pm 2$

**NOW**

$a = 2$

$b = \frac{-2}{a}$

$= \frac{-2}{2}$

$= -1$

**HENCE**

$\sqrt{3 - 4i} = 2 - i$

$a = -2$

$b = \frac{-3}{a}$

$= \frac{-2}{-2}$

$= 1$

**HENCE**

$\sqrt{3 - 4i} = -2 + i$

$\pm(2 - i)$

**04.** $\sqrt{7 + 24i}$

$\sqrt{7 + 24i} = a + bi$

$7 + 24i = (a + bi)^2$

$7 + 24i = a^2 + 2abi + b^2i^2$

$7 + 24i = a^2 + 2abi - b^2$

$7 + 24i = a^2 - b^2 + 2abi$

**COMPARING**

$a^2 - b^2 = 7$ …….. (1)

$2ab = 24$

$ab = 12$

$b = \frac{12}{a}$

subs in (1)

$a^2 - \left(\frac{12}{a}\right)^2 = 7$

$a^2 - \frac{144}{a^2} = 7$

$\frac{a^4 - 144}{a^2} = 7$

$a^4 - 144 = 7a^2$

$a^4 - 7a^2 - 144 = 0$

$(a^2 - 16)(a^2 + 9) = 0$

$a^2 = 16$

$a = \pm 4$

**NOW**

$a = 4$

$b = \frac{12}{a}$

$= \frac{12}{4}$

$= 3$

**HENCE**

$\sqrt{7 + 24i} = 4 + 3i$

$a = -4$

$b = \frac{12}{a}$

$= \frac{12}{-4}$

$= -3$

**HENCE**

$\sqrt{7 + 24i} = -4 - 3i$

$\pm(4 + 3i)$

**05.** $\sqrt{1 + 4\sqrt{3}i}$

$\sqrt{1 + 4\sqrt{3}i} = a + bi$

$1 + 4\sqrt{3}i = (a + bi)^2$

$1 + 4\sqrt{3}i = a^2 + 2abi + b^2i^2$

$1 + 4\sqrt{3}i = a^2 + 2abi - b^2$

$1 + 4\sqrt{3}i = a^2 - b^2 + 2abi$

**COMPARING**

$a^2 - b^2 = 1$ …….. (1)

$2ab = 4\sqrt{3}$

$ab = 2\sqrt{3}$

$b = \frac{2\sqrt{3}}{a}$

subs in (1)

$a^2 - \left(\frac{2\sqrt{3}}{a}\right)^2 = 1$

$a^2 - \frac{12}{a^2} = 1$

$\frac{a^4 - 12}{a^2} = 1$

$a^4 - 12 = a^2$

$a^4 - a^2 - 12 = 0$

$(a^2 - 4)(a^2 + 1) = 0$

$a^2 = 4$

$a = \pm 2$

**NOW**

$a = 2$

$b = \frac{2\sqrt{3}}{a}$

$= \frac{2\sqrt{3}}{2}$

$= \sqrt{3}$

**HENCE**

$\sqrt{-15 - 8i}$

$= 2 + \sqrt{3}i$

$a = -1$

$b = \frac{2\sqrt{3}}{a}$

$= \frac{2\sqrt{3}}{-2}$

$= -\sqrt{3}$

**HENCE**

$\sqrt{-15 - 8i}$

$= -2 - \sqrt{3}i$

$\pm(2 + \sqrt{3}i)$

**06.** $\sqrt{6 + 8i}$

$\sqrt{6 + 8i} = a + bi$

$6 + 8i = (a + bi)^2$

$6 + 8i = a^2 + 2abi + b^2i^2$

$6 + 8i = a^2 + 2abi - b^2$

$6 + 8i = a^2 - b^2 + 2abi$

**COMPARING**

$a^2 - b^2 = 6$ …….. (1)

$2ab = 8$

$ab = 4$

$b = \frac{4}{a}$

subs in (1)

$a^2 - \left(\frac{4}{a}\right)^2 = 6$

$a^2 - \frac{16}{a^2} = 6$

$\frac{a^4 - 16}{a^2} = 6$

$a^4 - 16 = 6a^2$

$a^4 - 6a^2 - 16 = 0$

$(a^2 - 8)(a^2 + 2) = 0$

$a^2 = 8$

$a = \pm 2\sqrt{2}$

**NOW**

$a = 2\sqrt{2}$

$b = \frac{4}{a}$

$= \frac{4}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \sqrt{2}$

**HENCE**

$\sqrt{3 - 4i}$

$= 2\sqrt{2} + \sqrt{2}i$

$= \sqrt{2}.(2 + i)$

$a = -2\sqrt{2}$

$b = \frac{4}{a}$

$= \frac{4}{-2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= -\sqrt{2}$

**HENCE**

$\sqrt{3 - 4i}$

$= -2\sqrt{2} - \sqrt{2}i$

$= -\sqrt{2}.(2 + i)$

$\pm\sqrt{2}(2 + i)$

**07.** $\sqrt{2(1 - \sqrt{3}i)}$

$\sqrt{2 - 2\sqrt{3}i} = a + bi$

$2 - 2\sqrt{3}i = (a + bi)^2$

$2 - 2\sqrt{3}i = a^2 + 2abi + b^2i^2$

$2 - 2\sqrt{3}i = a^2 + 2abi - b^2$

$2 - 2\sqrt{3}i = a^2 - b^2 + 2abi$

**COMPARING**

$a^2 - b^2 = 2$ …….. (1)

$2ab = -2\sqrt{3}$

$ab = -\sqrt{3}$

$b = \dfrac{-\sqrt{3}}{a}$

subs in (1)

$a^2 - \left(\dfrac{-\sqrt{3}}{a}\right)^2 = 2$

$a^2 - \dfrac{3}{a^2} = 2$

$\dfrac{a^4 - 3}{a^2} = 2$

$a^4 - 3 = 2a^2$

$a^4 - 2a^2 - 3 = 0$

$(a^2 - 3)(a^2 + 1) = 0$

$a^2 = 3$

$a = \pm \sqrt{3}$

**NOW**

$a = \sqrt{3}$

$b = \dfrac{-\sqrt{3}}{a}$

$= \dfrac{-\sqrt{3}}{\sqrt{3}}$

$= -1$

**HENCE**

$\sqrt{2 - 2\sqrt{3}i}$

$= \sqrt{3} - i$

$a = -\sqrt{3}$

$b = \dfrac{-\sqrt{3}}{a}$

$= \dfrac{-\sqrt{3}}{-\sqrt{3}}$

$= 1$

**HENCE**

$\sqrt{2 - 2\sqrt{3}i}$

$= -\sqrt{3} + i$

$\pm(\sqrt{3} - \sqrt{i})$

**08.** $\sqrt{3 + 2\sqrt{10}i}$

$\sqrt{3 + 2\sqrt{10}i} = a + bi$

$3 + 2\sqrt{10}i = (a + bi)^2$

$3 + 2\sqrt{10}i = a^2 + 2abi + b^2i^2$

$3 + 2\sqrt{10}i = a^2 + 2abi - b^2$

$3 + 2\sqrt{10}i = a^2 - b^2 + 2abi$

**COMPARING**

$a^2 - b^2 = 3$ …….. (1)

$2ab = 2\sqrt{10}$

$ab = \sqrt{10}$

$b = \dfrac{\sqrt{10}}{a}$

subs in (1)

$a^2 - \left(\dfrac{\sqrt{10}}{a}\right)^2 = 3$

$a^2 - \dfrac{10}{a^2} = 3$

$\dfrac{a^4 - 10}{a^2} = 3$

$a^4 - 10 = 3a^2$

$a^4 - 3a^2 - 10 = 0$

$(a^2 - 5)(a^2 + 2) = 0$

$a^2 = 5$

$a = \pm \sqrt{5}$

**NOW**

$a = \sqrt{5}$

$b = \dfrac{\sqrt{10}}{a}$

$= \dfrac{\sqrt{10}}{\sqrt{5}}$

$= \sqrt{2}$

**HENCE**

$\sqrt{3 + 2\sqrt{10}i}$

$= \sqrt{5} + \sqrt{2}i$

$a = -\sqrt{5}$

$b = \dfrac{\sqrt{10}}{a}$

$= \dfrac{\sqrt{10}}{-\sqrt{5}}$

$= -\sqrt{2}$

**HENCE**

$\sqrt{3 + 2\sqrt{10}i}$

$= -\sqrt{5} - \sqrt{2}i$

$\pm(\sqrt{5} + \sqrt{2}i)$

**09.** $\sqrt{2i}$

$$\sqrt{0 + 2i} = a + bi$$

$$0 + 2i = (a + bi)^2$$

$$0 + 2i = a^2 + 2abi + b^2i^2$$

$$0 + 2i = a^2 + 2abi - b^2$$

$$0 + 2i = a^2 - b^2 + 2abi$$

**COMPARING**

$$a^2 - b^2 = 0 \quad \ldots\ldots (1)$$

$$2ab = 2$$

$$ab = 1$$

$$b = \frac{1}{a}$$

subs in (1)

$$a^2 - \left(\frac{1}{a}\right)^2 = 0$$

$$a^2 - \frac{1}{a^2} = 0$$

$$\frac{a^4 - 1}{a^2} = 0$$

$$a^4 - 1 = 0$$

$$(a^2 - 1)(a^2 + 1) = 0$$

$$a^2 = 1$$

$$a = \pm 1$$

**NOW**

$$a = 1$$

$$b = \frac{1}{a} = \frac{1}{1} = 1$$

**HENCE**

$$\sqrt{2i} = 1 + i$$

$$a = -1$$

$$b = \frac{1}{a} = \frac{1}{-1} = -1$$

**HENCE**

$$\sqrt{2i} = -1 - i = -(1 + i)$$

$$\pm(1 + i)$$

**10.** $\sqrt{3i}$

$$\sqrt{0 + 3i} = a + bi$$

$$0 + 3i = (a + bi)^2$$

$$0 + 3i = a^2 + 2abi + b^2i^2$$

$$0 + 3i = a^2 + 2abi - b^2$$

$$0 + 3i = a^2 - b^2 + 2abi$$

**COMPARING**

$$a^2 - b^2 = 0 \quad \ldots\ldots (1)$$

$$2ab = 3$$

$$2ab = 3$$

$$b = \frac{3}{2a}$$

subs in (1)

$$a^2 - \left(\frac{3}{2a}\right)^2 = 0$$

$$a^2 - \frac{9}{4a^2} = 0$$

$$\frac{4a^4 - 9}{4a^2} = 0$$

$$4a^4 - 9 = 0$$

$$(2a^2 - 3)(2a^2 + 3) = 0$$

$$2a^2 = 3$$

$$a = \pm \sqrt{3}/\sqrt{2}$$

**NOW**

$$a = \sqrt{3}/\sqrt{2}$$

$$b = \frac{3}{2a} = \frac{3}{2 \cdot \frac{\sqrt{3}}{\sqrt{2}}} = \frac{\sqrt{3}}{\sqrt{2}}$$

**HENCE**

$$\sqrt{3i} = \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{3}i}{\sqrt{2}}$$

$$a = -\sqrt{3}/\sqrt{2}$$

$$b = \frac{3}{2a} = \frac{3}{2 \cdot \frac{-\sqrt{3}}{\sqrt{2}}} = \frac{-\sqrt{3}}{\sqrt{2}}$$

**HENCE**

$$\sqrt{3i} = \frac{-\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{3}i}{\sqrt{2}}$$

Polar form of z = a + bi is given as

$Z = r(\cos\theta + i\sin\theta)$

where

1. $a = r\cos\theta$ ; $b = r\sin\theta$

2. $r = \sqrt{a^2 + b^2}$
is called the MODULUS of the complex number z and is denoted by |z|

3. $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ is called the argument or amplitude of z

**NOTE I**

a) if $a > 0$ , $b > 0$ , then P(a,b) lies in the first quadrant , $0 < \theta < {}^{\pi}/2$

b) if $a < 0$ , $b > 0$ , then P(a,b) lies in the second quadrant , ${}^{\pi}/2 < \theta < \pi$

c) if $a < 0$ , $b < 0$ , then P(a,b) lies in the third quadrant , $\pi < \theta < {}^{3\pi}/2$

d) if $a > 0$ , $b < 0$ , then P(a,b) lies in the fourth quadrant , ${}^{3\pi}/2 < \theta < 2\pi$

**NOTE II**

a) if $a > 0$ , $b = 0$ , then P(a,b) lies on the positive direction of x – axis , amp(z) = 0

b) if $a < 0$ , $b = 0$ , then P(a,b) lies on the negative direction of x – axis , amp(z) = $\pi$

c) if $a = 0$ , $b > 0$ , then P(a,b) lies on the positive direction of y – axis , amp(z) = ${}^{\pi}/2$

d) if $a = 0$ , $b < 0$ , then P(a,b) lies on the negative direction of y – axis , amp(z) = ${}^{3\pi}/2$

01. 1 + i

$z = 1 + i$

$a = 1$ ; $b = 1$

MODULUS $r = \sqrt{a^2 + b^2}$

$= \sqrt{1 + 1}$

$= \sqrt{2}$

AMPLITUDE ($\theta$)

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{1}{1}\right)$

$= \tan^{-1}(1)$

$= 45^o$

$= {}^{\pi}/4$

POLAR FORM OF Z

$z = r(\cos\theta + i\sin\theta)$

$= \sqrt{2}\left(\cos{}^{\pi}/4 + i\sin{}^{\pi}/4\right)$

02. $4 + 4\sqrt{3}\, i$

$z = 4 + 4\sqrt{3}\, i$

$a = 4$ ; $b = 4\sqrt{3}$

MODULUS $r = \sqrt{a^2 + b^2}$

$= \sqrt{16 + 48}$

$= \sqrt{64}$

$= 8$

AMPLITUDE ($\theta$)

$a > 0$ , $b > 0$ $\therefore$ $\theta$ lies in the I QUADRANT

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$

$= \tan^{-1}(\sqrt{3})$

$= 60^o = {}^{\pi}/3$

POLAR FORM OF Z

$$z = r(\cos\theta + i\sin\theta)$$

$$= 8\left(\cos {}^{\pi}/_{3} + i\sin {}^{\pi}/_{3}\right)$$

03. $\dfrac{1+\sqrt{3}\,i}{2}$

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2}\,i$$

$$a = \frac{1}{2}\ ;\quad b = \frac{\sqrt{3}}{2}$$

MODULUS $\quad r = \sqrt{a^2 + b^2}$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= \sqrt{1}$$

$$= 1$$

AMPLITUDE ($\theta$)

$a > 0\ ,\ b > 0 \quad \therefore\ \theta$ lies in the I QUADRANT

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right)$$

$$= \tan^{-1}\left(\sqrt{3}\right)$$

$$= 60^{o}$$

$$= {}^{\pi}/_{3}$$

POLAR FORM OF Z

$$z = r(\cos\theta + i\sin\theta)$$

$$= 1\left(\cos {}^{\pi}/_{3} + i\sin {}^{\pi}/_{3}\right)$$

04. $\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\,i$

$$z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\,i$$

$$a = \frac{1}{\sqrt{2}}\ ;\quad b = \frac{1}{\sqrt{2}}$$

MODULUS $\quad r = \sqrt{a^2 + b^2}$

$$= \sqrt{\frac{1}{2} + \frac{1}{2}}$$

$$= \sqrt{1}$$

$$= 1$$

AMPLITUDE ($\theta$)

$a > 0\ ,\ b > 0 \quad \therefore\ \theta$ lies in the I QUADRANT

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \tan^{-1}\left(\frac{1/\sqrt{2}}{1/\sqrt{2}}\right)$$

$$= \tan^{-1}\left(1\right)$$

$$= 45^{o}$$

$$= {}^{\pi}/_{4}$$

POLAR FORM OF Z

$$z = r(\cos\theta + i\sin\theta)$$

$$= 1\left(\cos {}^{\pi}/_{4} + i\sin {}^{\pi}/_{4}\right)$$

05. $-1 + \sqrt{3}i$

$$z = -1 + \sqrt{3}i$$

$$a = -1\ ;\quad b = \sqrt{3}$$

MODULUS $\quad r = \sqrt{a^2 + b^2}$

$$= \sqrt{1 + 3}$$

$$= \sqrt{4}$$

$$= 2$$

AMPLITUDE ($\theta$)

$a < 0\ ,\ b > 0 \quad \therefore\ \theta$ lies in the II QUADRANT

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$$

$$= \tan^{-1}\left(-\sqrt{3}\right)$$

$$= 120^{o}$$

$$= {}^{2\pi}/_{3}$$

POLAR FORM OF Z

$$z = r\,(\cos\theta + i\sin\theta)$$

$$= 2\left(\cos {}^{2\pi}/_{3} + i \sin {}^{2\pi}/_{3}\right)$$

06. $\dfrac{1 + 2i}{1 - 3i}$

$$z = \frac{1 + 2i}{1 - 3i} \times \frac{1 + 3i}{1 + 3i}$$

$$= \frac{1 + 3i + 2i + 6i^2}{1 - 9i^2}$$

$$= \frac{1 + 5i - 6}{1 + 9}$$

$$= \frac{-5 + 5i}{10}$$

$$= \frac{-5}{10} + \frac{5\,i}{10}$$

$$= \frac{-1}{2} + \frac{1}{2}\,i$$

$$a = \frac{-1}{2} \; ; \quad b = \frac{1}{2}$$

MODULUS $r = \sqrt{a^2 + b^2}$

$$= \sqrt{\frac{1}{4} + \frac{1}{4}}$$

$$= \sqrt{{}^{1}/_{2}}$$

$$= \frac{1}{\sqrt{2}}$$

AMPLITUDE $(\theta)$

$a < 0$ , $b > 0$ $\therefore$ $\theta$ lies in the II QUADRANT

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \tan^{-1}\left(\frac{1/2}{-1/2}\right)$$

$$= \tan^{-1}\left(-1\right)$$

$$= 135^{o}$$

$$= {}^{3\pi}/_{4}$$

POLAR FORM OF Z

$$z = r\,(\cos\theta + i\sin\theta)$$

$$= \frac{1}{\sqrt{2}}\left(\cos {}^{3\pi}/_{4} + i \sin {}^{3\pi}/_{4}\right)$$

07. $\dfrac{1 + 7i}{(2 - i)^2}$

$$z = \frac{1 + 7i}{4 - 4i + i^2}$$

$$= \frac{1 + 7i}{4 - 4i - 1}$$

$$= \frac{1 + 7i}{3 - 4i}$$

$$= \frac{1 + 7i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i}$$

$$= \frac{3 + 4i + 21i + 28i^2}{9 - 16i^2}$$

$$= \frac{3 + 25i - 28}{9 + 16}$$

$$= \frac{-25 + 25i}{25}$$

$$= -1 + 1i$$

$$a = -1 \; ; \quad b = 1$$

MODULUS $r = \sqrt{a^2 + b^2}$

$$= \sqrt{1 + 1}$$

$$= \sqrt{2}$$

$a < 0$ , $b > 0$ $\therefore$ $\theta$ lies in the II QUADRANT

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \tan^{-1}\left(\frac{1}{-1}\right)$$

$$= \tan^{-1}\left(-1\right)$$

$= 135^o$

$= {}^{3\pi}/_4$

POLAR FORM OF Z

$z = r ( \cos\theta + i \sin\theta )$

$= \sqrt{2} \left( \cos {}^{3\pi}/_4 + i \sin {}^{3\pi}/_4 \right)$

08. $\dfrac{1}{1 + i}$

$z = \dfrac{1}{1+i} \times \dfrac{1-i}{1-i}$

$= \dfrac{1-i}{1-i^2}$

$= \dfrac{1-i}{1+1}$

$= \dfrac{1-i}{2}$

$= \dfrac{1}{2} + -\dfrac{1}{2} i$

$a = \dfrac{1}{2}$ ; $b = \dfrac{-1}{2}$

MODULUS $r = \sqrt{a^2 + b^2}$

$= \sqrt{\dfrac{1}{4} + \dfrac{1}{4}}$

$= \sqrt{{}^1/_2}$

$= \dfrac{1}{\sqrt{2}}$

AMPLITUDE ($\theta$)

$a > 0$ , $b < 0$ $\therefore$ $\theta$ lies in the IV QUADRANT

$\theta = \tan^{-1} \left( \dfrac{b}{a} \right)$

$= \tan^{-1} \left( \dfrac{-1/2}{1/2} \right)$

$= \tan^{-1} (-1)$

$= 315^o$

$= {}^{7\pi}/_4$

POLAR FORM OF Z

$z = r ( \cos\theta + i \sin\theta )$

$= \dfrac{1}{\sqrt{2}} \left( \cos {}^{7\pi}/_4 + i \sin {}^{7\pi}/_4 \right)$

09. 2i

$z = 0 + 2i$

$a = 0$ ; $b = 2$

MODULUS $r = \sqrt{a^2 + b^2}$

$= \sqrt{0 + 4}$

$= 2$

AMPLITUDE ($\theta$)

Since $a = 0$ , $b > 0$ , P(a,b) lies on the positive direction of Y – axis

$\therefore$ amp(z) $= \theta = {}^{\pi}/_2$

POLAR FORM OF Z

$z = r ( \cos\theta + i \sin\theta )$

$= 2 \left( \cos {}^{\pi}/_2 + i \sin {}^{\pi}/_2 \right)$

10. $-3i$

$z = 0 - 3i$

$a = 0$ ; $b = -3$

MODULUS $r = \sqrt{a^2 + b^2}$

$= \sqrt{0 + 9}$

$= 3$

AMPLITUDE ($\theta$)

Since $a = 0$ , $b < 0$ , P(a,b) lies on the negative direction of Y – axis

$\therefore$ amp(z) $= \theta = {}^{3\pi}/_2$

POLAR FORM OF Z

$z = r ( \cos\theta + i \sin\theta )$

$= 3 \left( \cos {}^{3\pi}/_2 + i \sin {}^{3\pi}/_2 \right)$